

Higgs-Dilaton Lagrangian from Spectral Regularization

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Abstract

In this letter we calculate the full Higgs-Dilaton action describing the Weyl anomaly using the bosonic spectral action. This completes the work we started in our previous paper (JHEP 1110 (2011) 001). We also clarify some issues related to the dilaton and its role as collective modes of fermions under bosonization.

Keywords: Spectral action, Higgs-dilaton couplings, Weyl anomaly

1 Introduction

In [1, 2] we discussed the idea of bosonization of all Standard Model fermion degrees of freedom into a single collective scalar field and we computed the effective Higgs dilation potential, expressing it via the bosonic spectral action [3]. The computation was performed for constant fields and assuming a space-time to be flat and the dilaton to be spatially constant. This gives the main features of the potential, but does not describe completely the action since all derivative terms, including the kinetic term, were absent from the analysis. In this letter we present the complete calculation, to render the model implementable for investigation of the cosmic evolution.

The presence of the dilaton field comes out from a spectral or finite mode regularization (FMR) and the anomaly bosonization. In [4–7] the mentioned approach was introduced and applied for the axial anomaly and in [8] for the Weyl anomaly in case of massless fermions.* The principle is based on the regularization of a partition function of a fermionic field theory whose fermionic action is generically written as

$$S_F = \langle \Psi | D | \Psi \rangle = \int d^4x \sqrt{g} \bar{\psi}(x) D_x \psi(x) \quad (1.1)$$

We will call the operator D the generalized Dirac operator, or just the Dirac operator. It is a matricial operator which contains the usual Dirac operator, but also other terms, such as the mass of the particles. The role of this operator is crucial in the noncommutative geometry framework [12]. The action (1.1) once inserted in a functional interaction integration diverges, and should be regularized, and the FMR, that counts only the eigenvalues of D smaller than a cutoff scale Λ , spoils a Weyl invariance of the partition function.

However in [13] it was shown that one can restore the Weyl invariance via an addition to the classical action a slight variant of the spectral action which includes the dilaton. In contrast to the approach [13] and [14], in the present approach the dilaton is not considered as an independent field but is constructed from the Standard Model fermions via the anomalous bosonization. Investigation on the consequences of the spectral action for cosmology [15–17] are of great interest and this paper can be a starting contribution for the role of the dilaton. Moreover the applicability of the letter goes beyond the noncommutative geometry aspects, because the data we use is just is the action for fermions of the kind of the one for the standard model.

The paper is organized as follows. In section 2 we show that the Weyl transformed generalized Dirac operator is nothing but the original operator computed with transformed vierbeins and Higgs fields. In section 3 we recall the idea of the Weyl anomaly bosonization and introduction of the dilaton field as a collective degree of freedom of all standard model

*All these consideration however requires a compact spacetime, but the principle of a cutoff on the momentum eigenvalues is more general, see for example [10, 11].

fermions. After that, using the observation from section 2 we finally compute all terms of the Higgs-dilaton potential in section 4. The last section contains conclusions. Several computational details are in two appendices.

2 Generalized Weyl invariance

In [2] we discussed the invariance properties of the fermionic action (1.1) under the following transformation:

$$D \rightarrow e^{-\frac{\phi}{2}} D e^{-\frac{\phi}{2}}, \quad \Psi \rightarrow e^{\frac{1}{2}\phi} \psi, \quad \text{or} \quad D_x \rightarrow e^{-\frac{5}{2}\phi} D_x e^{+\frac{3}{2}\phi}, \quad \psi \rightarrow e^{-\frac{3}{2}\phi} \psi, \quad (2.1)$$

where

$$D \equiv g^{-\frac{1}{4}} D_x g^{\frac{1}{4}} \quad \Psi \equiv g^{-\frac{1}{4}} \psi. \quad (2.2)$$

The operator D is the abstract operator on the Hilbert space of the vectors Ψ , while D_x and $\psi(x)$ are their realizations as differential operators and functions on spacetime respectively. The quantities ψ and D_x are usually used to write down the classical fermionic action with a diffeomorphic invariant coordinate measure $d^4x\sqrt{g}$, while the quantities Ψ and D are more convenient in the functional integral formalism, due to the diffeomorphic invariance of the measure $[d\Psi][d\bar{\Psi}]$ (see [9]).

The Dirac operator D acts on left-right spinors as

$$D_x = \begin{pmatrix} D_G & \gamma_5 \otimes S \\ \gamma_5 \otimes S^\dagger & D_G \end{pmatrix} \quad (2.3)$$

where D_G is a "geometric" part of the Dirac operator,[†]

$$D_G = ie_k^\mu \gamma^k \left(\partial_\mu - \frac{i}{2} \omega_\mu^{mn} \sigma_{mn} - i A_\mu^a T^a \right), \quad (2.4)$$

that contains the spin connection ω_μ^{mn} and gauge fields A_μ , and S contains the information about Higgs field H , Yukawa couplings, mixings i.e. all terms which couple the left and right spinors. The gravitational background is in general nontrivial. For the purpose of this note the relevant aspect is the presence on the Higgs field H , on which we concentrate our attention. For the following it is important to note that S is *linear* in H .

The important fact is that the transformation (2.1) is equivalent to:

$$g_{\mu\nu} \rightarrow e^{2\phi} g_{\mu\nu}, \quad \psi \rightarrow e^{-\frac{3}{2}\phi} \psi, \quad H \rightarrow e^{-\phi} H. \quad (2.5)$$

[†]Following a well established tradition, we use greek indexes to label coordinates, latin letters k, l, m, n for Lorentz indexes and a, b, c for gauge indexes.

The law of transformation of the Higgs field H is in agreement with (2.1). To prove the equivalence of (2.1) and (2.5) finally, we notice, that, under[‡] (2.5), the geometric part of the Dirac operator transforms as follows:

$$D_G \rightarrow e^{-\frac{5\phi(x)}{2}} D_G e^{\frac{3\phi(x)}{2}}. \quad (2.6)$$

The mentioned result is present in [9], however in the appendix A we give a more detailed proof.

We also remark that we *do not* transform the gauge fields A_μ^a : they appear in D_G multiplied by e_k^μ , so the correct transformation of the "gauge term" of D_G is automatically provided by the transformation of the vierbeins.

3 Generalized Weyl anomaly bozonization

Following [2] we consider fermions, in a fixed gauge, Higgs and gravity background. Due to a generalized[§] Weyl anomaly, the fermionic partition function

$$Z_F = \int [d\Psi][d\bar\Psi] e^{-S_F[\bar\Psi, \Psi, \text{bosonic background}]} \quad (3.1)$$

is not invariant under (2.5). In [2] it was shown that the Weyl non invariant part of Z_F can be expressed in terms of the collective degree of freedom of all SM fermions, the dilation:

$$\int [d\Psi][d\bar\Psi] e^{-S_F[\bar\Psi, \Psi, \text{bosonic background}]} = \int [d\phi] e^{-S_{coll}[\phi, \text{bosonic background}] + W_{inv}}, \quad (3.2)$$

where W_{inv} is (nonlocal) Weyl invariant functional of background fields, and S_{coll} is a *local* functional of background fields and the dilation ϕ . For a flat space-time and coordinate independent fields S_{coll} was computed in [2].

Let us clarify some aspects of the introduction of the collective degree of freedom of all fermions, or bosonization. In our context the term "bosonisation" does not mean that some composite operator $O_\phi(x)$, constructed from the scalar field ϕ and its derivatives, equals another composite operator $O_\Psi(x)$, constructed from the fermionic fields Ψ and $\bar\Psi$. More generally it means that the vacuum expectation of the product of n bosonic composite operators $O_\phi(x)$ equals the vacuum expectation of the product of n fermionic

[‡]More carefully one should write the corresponding transformation of vierbeins instead of the transformation of a metric tensor.

[§]Standard Weyl anomaly is related with the transformation of a metric tensor and fermions. Since now we also transform the Higgs field H , now should we call the anomaly "generalized". In the following for brevity we will skip the word "generalized".

composite operators $O_\Psi(x)$ for $n = 1, 2, \dots$, i.e. equality of corresponding classes of Green functions.

$$\langle O_\Psi(x_1), \dots, O_\Psi(x_n) \rangle_{\text{ferm. vacuum}} = \langle O_\phi(x_1), \dots, O_\phi(x_n) \rangle_{\text{bos. vacuum}}, \quad n = 1, 2, \dots \quad (3.3)$$

Now we will specify the mentioned classes of Green functions. Substitute $g_{\mu\nu} = e^{2\alpha} g_{\mu\nu}$ and $H = e^{-\alpha} H$ in (3.2) and consider α as a source. Since the invariant part W_{inv} in the right hand side of (3.2) remains unchanged under this substitution, it will not give contribution, one has:

$$\left(\frac{\delta^n}{\delta\alpha(x_1) \dots \delta\alpha(x_n)} \log Z_F^\alpha \right) \Big|_{\alpha_1, \dots, \alpha_n=0} = \left(\frac{\delta^n}{\delta\alpha(x_1) \dots \delta\alpha(x_n)} \log Z_{coll}^\alpha \right) \Big|_{\alpha_1, \dots, \alpha_n=0}, \quad (3.4)$$

where

$$Z_F^\alpha \equiv \int [d\Psi][d\bar\Psi] e^{-S_F[\bar\Psi, \Psi, e^{2\alpha} g_{\mu\nu}, e^{-\alpha} H]}, \quad Z_{coll}^\alpha \equiv \int [d\phi] e^{-S_{coll}[\phi, e^{2\alpha} g_{\mu\nu}, e^{-\alpha} H]} \quad (3.5)$$

In our case the composite fermionic operator O_Ψ , that we bosonize, and the corresponding bosonic operator O_ϕ are given correspondingly by:

$$O_\Psi(x) = \left(\frac{\delta}{\delta\alpha(x)} S_F[\bar\Psi, \Psi, e^{2\alpha} g_{\mu\nu}, e^{-\alpha} H] \right)_{\alpha=0}, \quad (3.6)$$

$$O_\phi(x) = \left(\frac{\delta}{\delta\alpha(x)} S_{coll}[\phi, e^{2\alpha} g_{\mu\nu}, e^{-\alpha} H] \right)_{\alpha=0}. \quad (3.7)$$

Notice that in the absence of the Higgs field, $H = 0$, these operators are nothing but (up to a \sqrt{g} factor) traces of corresponding stress energy tensors $T_{F, coll}^{\mu\nu}(x) = \frac{2\delta}{\sqrt{g}\delta g_{\mu\nu}(x)} S_{F, coll}$. It is remarkable, that in this case the classical $T_{\mu F}^\mu$ vanishes on the equations of motion, however the quantum vacuum average $\langle T_{\mu F}^\mu(x) \rangle_{\text{ferm. vac.}} \neq 0$, due to the famous trace anomaly (see e.g. [9]). The collective action describes the trace anomaly already on classical level:

$$\langle T_{\mu F}^\mu(x) \rangle_{\text{ferm. vac.}} = \langle T_{\mu coll}^\mu(x) \rangle_{\text{bos. vac.}} \simeq T_{\mu coll}^\mu(x) \Big|_{\phi=\phi_{class}} + \text{loop corrections}, \quad (3.8)$$

where $\phi_{class}(x)$ solves the classical equations of motion $\frac{\delta S_{coll}[\phi]}{\delta\phi(x)} = 0$. In contrast to the fermionic partition function, the bosonic partition function doesn't poses the trace anomaly, and the Weyl non invariance of action appears already at classical level.

In the presence of the Higgs field, i.e. when the Dirac operator is given by (2.3), the operator $O_\Psi(x)$, given by (3.6) equals to

$$O_\Psi = \sqrt{g} (T_{\mu F}^\mu - \gamma_5 \otimes S(H) \bar\psi \psi), \quad T_{\mu F}^\mu \equiv \frac{2g_{\mu\nu}}{\sqrt{g}} \frac{\delta}{\delta g_{\mu\nu}} S_F, \quad (3.9)$$

besides $\langle T_\mu^\mu \rangle$ now $\langle O_\Psi \rangle$ contains an additional fermionic condensate $\langle \bar{\psi}(x)\psi(x) \rangle$ contribution.

In the next section we will evaluate the collective action S_{coll} expressing it through the (modified) bosonic spectral action. This computation is strongly based on the use of the heat-kernel expansion, that being an asymptotic expansion, strictly makes sense in the weak fields approximation, and faces problems beyond it [18]. Nevertheless the bosonization that we discuss is also valid in low energy region, that justifies the use of the heat kernel in our treatment.

4 Computation of Higgs-dilaton Action.

Following our previous work, and taking into account that now the dilaton field depends on space-time coordinates, we have:

$$S_{coll} = - \left(1 - \Lambda^2 \log \frac{\Lambda^2}{\mu^2} \partial_{\Lambda^2} \right) \int_0^1 dt \text{Tr} \left\{ \phi \chi \left(\frac{(e^{-\frac{\phi t}{2}} D e^{-\frac{\phi t}{2}})^2}{\Lambda^2} \right) \right\} \quad (4.1)$$

Since we perform the finite mode regularization, that cuts all eigenvalues of D higher than the cutoff scale Λ , we actually deal with a *bounded* operator, therefore under the sign of Tr we can replace $\left(\tilde{D} \right)_{\phi t} \equiv e^{-\frac{\phi t}{2}} D e^{-\frac{\phi t}{2}}$ by $\left(\tilde{D}_x \right)_{\phi t} \equiv e^{-\frac{5}{2}\phi t} D_x e^{+\frac{3}{2}\phi t}$, where $D_x \equiv g^{-\frac{1}{4}} D g^{\frac{1}{4}}$ (c.f.(1.1)). As shown in Sec. 2 we can rewrite (4.1) in the following way:

$$S_{coll} = - \left(1 - \Lambda^2 \log \frac{\Lambda^2}{\mu^2} \partial_{\Lambda^2} \right) \int_0^1 dt \left(\text{Tr} \left\{ \phi \chi \left(\frac{D_x^2}{\Lambda^2} \right) \right\} \right) \Big|_{g_{\mu\nu}=(\tilde{g}_{\mu\nu})_{\phi t}, H=(\tilde{H})_{\phi t}} \quad (4.2)$$

Using the heat kernel expansion one can show, that (for details see for example [3])

$$\begin{aligned} \text{Tr} \left\{ \phi \chi \left(\frac{D_x^2}{\Lambda^2} \right) \right\} &= \int d^4x \sqrt{g} \phi \left(\frac{45\Lambda^4}{8\pi^2} + \frac{15\Lambda^2}{16\pi^2} (R - 2y^2 H^2) \right. \\ &\quad + \frac{1}{4\pi^2} \left(\frac{3}{8} R_{;\mu}{}^\mu + \frac{11}{32} G_B - y^2 H_{;\mu}{}^\mu + 3y^2 \left(D_\mu H D^\mu H - \frac{1}{6} R H^2 \right) \right. \\ &\quad \left. \left. + 3z^2 H^4 + G_{\mu\nu}^i G^{\mu\nu i} + W_{\mu\nu}^\alpha W^{\mu\nu\alpha} + \frac{5}{3} B_{\mu\nu} B^{\mu\nu} - \frac{9}{16} C_{\mu\nu\rho\lambda} C^{\mu\nu\rho\lambda} \right) \right) \end{aligned} \quad (4.3)$$

where y^2 and z^2 stand for correspondingly quadratic and quartic combinations of the Yukawa couplings, whose precise definition can be found for example in [3, Eq. 3.17]. Since the Yukawa couplings are strongly dominated by the one of the top quark Y_t , one can keep in mind, that $y^2 \simeq Y_t^2$, $z^2 \simeq Y_t^4$. G_B denotes the Gauss-Bonnet density:

$$G_B \equiv \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} R_{\mu\nu}^{\alpha\beta} R_{\rho\sigma}^{\gamma\delta}. \quad (4.4)$$

Substituting the expression (4.3) into (4.2) we finally get the result:

$$\begin{aligned}
S_{coll} \equiv & \int d^4x \sqrt{g} \left(A (e^{4\phi} - 1) + BH^2 (e^{2\phi} - 1) - C\phi H^4 - \alpha_1 (e^{2\phi} - 1) R + \alpha_2 e^{2\phi} (\phi_{;\mu}\phi^{;\mu}) \right. \\
& - \alpha_3 \phi \left(3y^2 \left(D_\mu H D^\mu H - \frac{1}{6} R H^2 \right) + G_{\mu\nu}^i G^{\mu\nu i} + W_{\mu\nu}^\alpha W^{\mu\nu\alpha} + \frac{5}{3} B_{\mu\nu} B^{\mu\nu} - \frac{9}{16} C_{\mu\nu\rho\lambda} C^{\mu\nu\rho\lambda} \right) \\
& \left. - \alpha_4 \left(12R (\phi_{;\mu}^\mu + \phi_{;\mu}\phi^{;\mu}) + 11\phi G_B + 44G^{\mu\nu}\phi_{;\mu}\phi_{;\nu} + 14 (\phi_{;\mu}^\mu + \phi_{;\mu}\phi^{;\mu})^2 + 22 (\phi_{;\mu}^\mu)^2 \right) \right)
\end{aligned} \tag{4.5}$$

where $G_{\mu\nu}$ stands for the Einstein tensor and the constants $A, B, C, \alpha_1.. \alpha_4$, are defined as follows:

$$\begin{aligned}
A &= \left(2 \log \frac{\Lambda^2}{\mu^2} - 1 \right) \frac{45\Lambda^4}{32\pi^2}, \quad B = \left(1 - \log \frac{\Lambda^2}{\mu^2} \right) \frac{15\Lambda^2 y^2}{16\pi^2}, \quad C = \frac{3z^2}{4\pi^2}, \\
\alpha_1 &= \left(1 - \log \frac{\Lambda^2}{\mu^2} \right) \frac{15\Lambda^2}{32\pi^2}, \quad \alpha_2 = \left(1 - \log \frac{\Lambda^2}{\mu^2} \right) \frac{45\Lambda^2}{16\pi^2}, \quad \alpha_3 = \frac{1}{4\pi^2}, \quad \alpha_4 = \frac{1}{128\pi^2}.
\end{aligned} \tag{4.6}$$

For technical details of this calculation see the appendix

5 Conclusions

The full Higgs-dilaton Lagrangian is computed, in the general case, thus completing the program me started in [1, 2], the result given by the expression (4.5). We opens the gate for a complete analysis of the cosmological consequences of the model. We may already say that for a choice of the normalization point parameter μ , that provides existence of minimum of the Higgs-dilaton potential, kinetic term of the dilaton field comes out with the correct positive sign. In the local minimum point $\phi = \phi_m$ the induced R term appears also with the correct positive plus. We have also clarified the role of the bosonic fields vs. the fermionic ones, and given a prescription to recognize the bosonization.

A Equivalence of Transformations

In this appendix we give the details of the equivalence of the two transformations (2.1) and (2.5).

The Weyl transformation of the metric tensor in terms of vierbeins is given by:

$$e_{\mu k} \rightarrow e^{\phi(x)} e_{\mu k}, \quad e^{\mu k} \rightarrow e^{-\phi(x)} e^{\mu k}. \tag{A.1}$$

The spin connection ω_μ^{mn} has the following expression via vierbeins (see [9]):

$$\omega_\mu^{mn} = \frac{1}{2} e^{m\lambda} e^{n\rho} (C_{\lambda\rho\mu} - C_{\rho\lambda\mu} - C_{\mu\lambda\rho}), \tag{A.2}$$

where

$$C_{\lambda\rho\mu} = e_{\lambda}^k (\partial_{\rho} e_{k\mu} - \partial_{\mu} e_{k\rho}). \quad (\text{A.3})$$

Substituting the transformation (A.1) in (A.3) and (A.2) we find the following law of the spin connection transformation under (A.1):

$$\omega_{\mu}^{mn} \rightarrow \omega_{\mu}^{mn} + e_{\mu}^m e^{n\rho} \partial_{\rho} \phi - e_{\mu}^n e^{m\lambda} \partial_{\lambda} \phi. \quad (\text{A.4})$$

Now we remind, that the generators of the representation of Lorentz group σ_{mn} for spin 1/2 have the following form in terms of the Dirac matrixes:

$$\sigma_{mn} = \frac{i}{4} [\gamma_m, \gamma_n]. \quad (\text{A.5})$$

Therefore one has the following law of transformation the combination $\frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn}$ (this formula is presented in [9][¶]):

$$\frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn} \rightarrow \frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn} - \frac{1}{2} \gamma_{\mu} \gamma^{\alpha} \partial_{\alpha} \phi + \frac{1}{2} \partial_{\mu} \phi. \quad (\text{A.6})$$

So we have:

$$-\gamma^{\mu} \frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn} \rightarrow -\gamma^{mu} \frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn} + \frac{3}{2} \gamma^{\mu} \partial_{\mu} \phi. \quad (\text{A.7})$$

Finally, using (A.7) and (A.1) we obtain:

$$\begin{aligned} D_G &\equiv i e_k^{\mu} \gamma^k \left(\partial_{\mu} - \frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn} - i A_{\mu}^a T^a \right) \rightarrow \\ &\rightarrow i e_k^{\mu} \gamma^k e^{-\phi} \left(\partial_{\mu} - \frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn} - i A_{\mu}^a T^a + \frac{3}{2} \partial_{\mu} \phi \right) = \\ &= i e_k^{\mu} \gamma^k e^{-\frac{5\phi}{2}} \left(\partial_{\mu} - \frac{i}{2} \omega_{\mu}^{mn} \sigma_{mn} - i A_{\mu}^a T^a \right) e^{+\frac{3\phi}{2}} = e^{-\frac{5\phi}{2}} D_G e^{+\frac{3\phi}{2}}. \end{aligned} \quad (\text{A.8})$$

B Computational details.

B.1 R contribution.

Under the Weyl transformation of the metric tensor,

$$g_{\mu\nu} \rightarrow (g_{\mu\nu})_{\phi} \equiv e^{2\phi} g_{\mu\nu} \quad (\text{B.1})$$

the scalar curvature transforms as follows:

$$R \rightarrow \left(\tilde{R} \right)_{\phi} \equiv e^{-2\phi} \left(R + 6 \left(\phi_{;\mu}^{\mu} + \phi_{;\mu} \phi_{;\mu}^{\mu} \right) \right), \quad (\text{B.2})$$

and integrating by parts one can easily show, that

$$-\int d^4x \phi \int_0^1 dt \left(\sqrt{\tilde{g}} \tilde{R} \right)_{\phi t} = \int d^4x \sqrt{g} \left(-\frac{1}{2} (e^{2\phi} - 1) R + 3 \cdot e^{2\phi} \left(\phi_{;\mu} \phi_{;\mu}^{\mu} \right) \right). \quad (\text{B.3})$$

[¶]Comparing our and [9] formulas: note that Fujikawa and Suzuki use $\alpha(x) = -\phi(x)$

B.2 $(H^2)_{;\mu}^{\mu}$ and $R_{;\mu}^{\mu}$ contributions.

For a scalar quantity f , that transforms under the Weyl transformation (B.1) as

$$f \rightarrow \left(\tilde{f}\right)_{\phi}, \quad (\text{B.4})$$

its Laplacian $\Delta f \equiv \nabla_{\mu} \nabla^{\mu} f$ transforms as follows:

$$\Delta f \rightarrow \left(\tilde{\Delta} \tilde{f}\right)_{\phi} = e^{-4\phi} \nabla^{\mu} e^{2\phi} \nabla_{\mu} \left(\tilde{f}\right)_{\phi}. \quad (\text{B.5})$$

For $f = H^2$ and $f = R$, using the transformation law for H that can be inferred from (2.1), and relations (B.1) and (B.5), we obtain the following contributions to the Higgs-dilaton potential:

$$- \int d^4x \phi \int_0^1 dt \sqrt{\tilde{g}} \left(\tilde{\Delta} \tilde{H}^2\right)_{\phi t} = - \int d^4x \sqrt{g} \left(\phi_{;\mu}^{\mu} + \phi_{;\mu} \phi_{;\mu}^{\mu}\right) H^2 \quad (\text{B.6})$$

and

$$- \int d^4x \phi \int_0^1 dt \left(\sqrt{\tilde{g}} \tilde{\Delta} \tilde{R}\right)_{\phi t} = - \int d^4x \sqrt{g} \left(\left(\phi_{;\mu}^{\mu} + \phi_{;\mu} \phi_{;\mu}^{\mu}\right) R + 3 \left(\phi_{;\mu}^{\mu} + \phi_{;\mu} \phi_{;\mu}^{\mu}\right)^2\right). \quad (\text{B.7})$$

B.3 G_B contribution.

It is known from the differential geometry, that the Gauss-Bonnet density G_B in a four dimensional space-time can be presented in the following form, convenient for the forthcoming analysis:

$$G_B = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - 2 \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2\right). \quad (\text{B.8})$$

For the transformed Ricci tensor under the Weyl transformation (B.1) we have:

$$\left(\tilde{R}_{\mu\nu}\right)_{\phi} = R_{\mu\nu} + 2 \left(\phi_{;\mu\nu} - \phi_{;\mu} \phi_{;\nu}\right) + \left(\phi_{;\lambda}^{\lambda} + 2\phi_{;\lambda} \phi_{;\lambda}^{\lambda}\right) g_{\mu\nu} \quad (\text{B.9})$$

Using laws of transformations of the Ricci tensor and the scalar curvature (B.9), (B.2) and also Weyl invariance of the Weyl tensor contribution after some simple computations we obtain:

$$\sqrt{g} G_B \rightarrow \left(\sqrt{\tilde{g}} \tilde{R}^* \tilde{R}^*\right)_{\phi} = \sqrt{g} \left(G_B + \nabla_{\mu} J^{\mu}\right). \quad (\text{B.10})$$

where the current J^{μ} is defined as follows:

$$J^{\mu} \equiv 8 \left(-\phi_{;\nu} G^{\nu\mu} + \left(\phi_{;\lambda}^{\lambda} + \phi_{;\lambda} \phi_{;\lambda}^{\lambda}\right) \phi_{;\mu}^{\mu}\right) - 4 \left(\phi_{;\lambda} \phi_{;\lambda}^{\lambda}\right)_{;\mu}^{\mu}, \quad (\text{B.11})$$

Contribution of the Gauss-Bonnet term to the Higgs-dilaton potential is propotional (with the sign plus) to the following expression:

$$- \int d^4x \phi(x) \int_0^1 dt \left(\sqrt{\tilde{g}} \tilde{R}^* \tilde{R}^* \right)_{\phi t} = \int d^4x \sqrt{g} \left(-\phi G_B - 4G^{\mu\nu} \phi_{;\mu} \phi_{;\nu} + 2 \left(\phi_{;\mu} \phi_{;\mu} \right)^2 + 4 \left(\phi_{;\mu} \phi_{;\mu} \right) \phi_{;\lambda}^{\lambda} \right). \quad (\text{B.12})$$

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